

BOOK REMASTER · RESTORATION SAMPLE

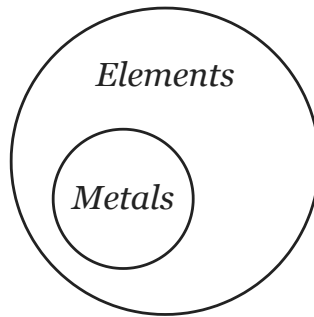
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tion. This is best indicated by a kind of diagram, first used by the celebrated mathematician Euler, in his letters to a German princess. In Fig. 1, the metals are supposed to be enclosed in the small circle somewhat as sheep might be in a pincfold, this circle containing all the metals and nothing else. The greater circle is supposed to contain in a similar manner all the elements and nothing else. Now as the small circle is wholly within the larger one, it follows that all the metals must be counted as

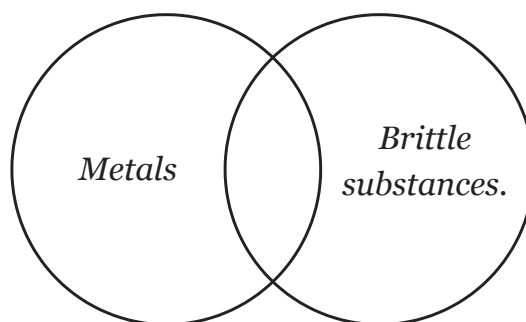
Fig. 1.



elements, but of the part of the elements outside the circle of metals we know nothing from the proposition.

The **particular affirmative proposition I** exactly resembles A in meaning, except that only part of the subject is brought into question. When I say that “some metals are brittle,” I mean that of a collection of all the different metals a few at least might be picked out which would be found to be brittle; but the word *some* is exceedingly indefinite, shewing neither the exact number of brittle metals, nor how we are to know them from the others, unless indeed by trying whether they are brittle. This proposition will be properly represented in Euler’s mode by two intersecting circles, one supposed to enclose all metals, and the other all brittle substances. The mere fact of the two circles intersecting proves that some

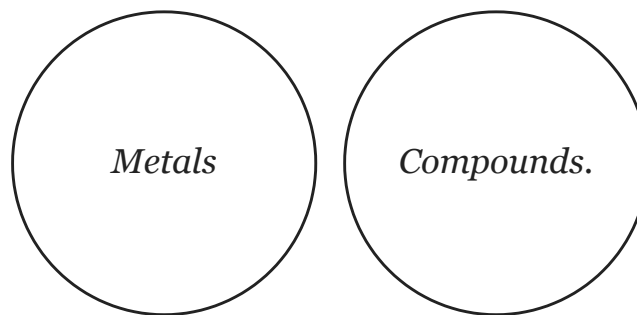
Fig. 2.



part of one class must coincide with some part of the other class, which is what the proposition is intended to express. Concerning the portions of the circles which do not overlap the proposition tells us nothing.

The **universal negative proposition E** denies the existence of any agreement or coincidence between the subject and predicate. Thus from “no metals are compound substances,” we learn that no metal is to be found among compound substances, and it follows necessarily that no compound substance can be found among the metals. For were there a compound substance among the metals, there would evidently be one metal at least among the compound substances. This entire separation in thought of the two classes is well shewn in Euler’s method by two disconnected circles.

Fig. 3.



The reader will easily see that the proposition E is distinguished from A and I, by the fact that it gives us some information concerning the *whole of the predicate*, because we learn that none of the objects included in the predicate can be found among those included in the subject. The affirmative propositions, on the other hand, warranted us in holding that the objects denoted by the subject, or some particular part of them, were included in the predicate, but *they give us no warrant* for saying that any specified part of the predicate is in the subject. Because we merely know that a substance is an element, we do not learn from the proposition “all metals are elements” whether it is a metal or not. And from the proposition “some metals are brittle,” we certainly cannot ascertain whether any particular brittle substance is a metal. We must seek the information from other sources. But from “no metals are compounds” we learn of any compound substance that it is not a metal, as well as of a metal that it is not a compound substance.

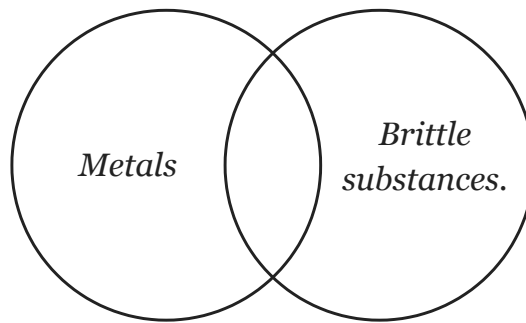
The important difference above explained is expressed in technical language by saying that the proposition E *distributes its predicate*, whereas the affirmative propositions A and I *do not distribute their predicates*. By **distribution** of a term is simply meant *taking it universally*, or *referring to all parts of it*; and as the validity of any argument or syllogism will usually depend upon the sufficient distribution of the terms occurring in it, too much attention cannot be paid to this point.

Judging from the examples we have had, it will be seen that the universal affirmative distributes its subject, but not its predicate; for it gives us some information concerning all metals, but not all elements. The particular affirmative distributes neither subject nor predicate; for we do not learn anything from our example concerning all metals nor concerning all brittle substances. But the universal negative distributes both subject and predicate, for we learn something of *all metals* and also of *all compound substances*.

The particular negative proposition O will be found to distribute its predicate, but not its subject. When I say *some metals are not brittle*, I intentionally refer only to a part of the metals, and exclude them from the class of *brittle substances*; but I cannot help at the same time referring to

the *whole* of the brittle substances. If the metals in question coincided with any part of the brittle substances they could not be said to be excluded from the class. To exclude a thing from any space, as from a particular chamber of a house, it must not merely be removed from some part, but from any part, or from the whole of that space or chamber. Euler's diagram for this proposition may be constructed in the same manner as for the proposition I as follows:—

Fig. 4.



It is apparent that though part of the metals fall into the circle of brittle substances, yet the remaining portion are excluded from any part of the predicate.

We may state the result at which we have now arrived in the following form:—

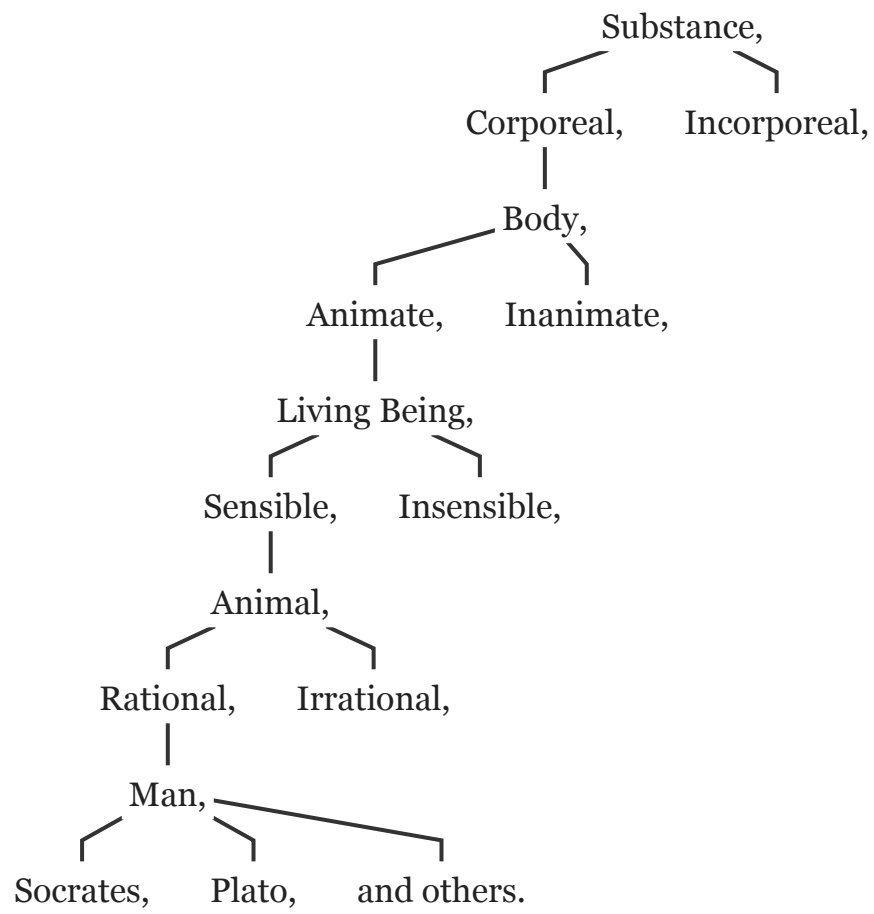
Proposition			Subject.	Predicate.
Universal	Affirmative	A.	Distributed.	Undistributed.
	Negative	E.	Distributed.	Distributed.
Particular	Affirmative	I.	Undistributed.	Undistributed.
	Negative	O.	Undistributed.	Distributed.

which belongs to the whole of the genus, from the **Specific Property**, which belongs to the whole of a *lowest* species.

Lastly, an **accident** (Latin *accidens*, Greek συμβεβηκός) is any quality which may indifferently belong or not belong to a class, as the case may be, without affecting the other qualities of the class. The word means that which *falls* or happens by chance, and has no necessary connection with the nature of a thing. Thus the absolute size of a triangle is a pure accident as regards its geometrical properties; for whether the side of a triangle be $\frac{1}{10}$ of an inch or a million miles, whatever Euclid proves to be true of one is true of the other. The birthplace of a man is an accident concerning him, as are also the clothes in which he is dressed, the position in which he rests, and so on. Some writers distinguish separable and inseparable accidents. Thus the clothes in which a man is dressed is a separable accident, because they can be changed, as can also his position, and many other circumstances; but his birthplace, his height, his Christian name, &c., are inseparable accidents, because they can never be changed, although they have no necessary or important relation to his general character.

As an illustration of some part of the scheme of classification described under the name of Predicables, I may here give, as is usual in manuals of Logic, the **Tree of Porphyry**, a sort of example of classification invented by one of the earliest Greek logicians, named Porphyrius. I have simplified the common form in which it is given by translating the Latin names and omitting superfluous words.

In this Tree we observe a succession of genera and species—Substance, Body, Living Being, Animal and Man. Of these Substance is the *summum genus*, because it is not regarded as a species of any higher class; Man is the *infima species*, because it is a class not divided into any lower class, but only into individuals, of whom it is

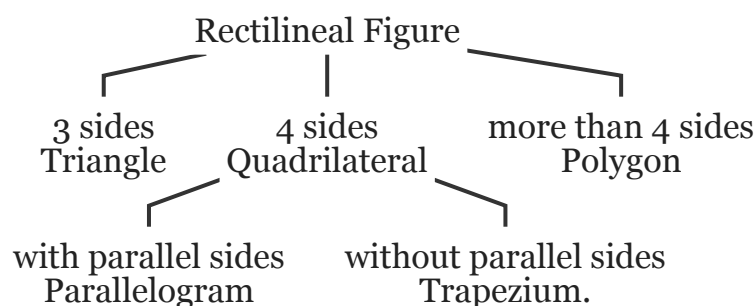


usual to specify Socrates and Plato. Body, Living Being, and Animal are called subaltern genera and species, because each is a species as regards the next higher genus, and a genus as regards the next lower species. The qualities implied in the adjectives Corporeal, Animate, Sensible (*i.e.* capable of feeling) and Rational are the successive differences which occasion a division of each genus into species. It will be evident that the negative parts of the genera, namely Incorporeal Substance, Inanimate Body, &c., are capable of subdivision, which has not been carried out in order to avoid confusing the figure.

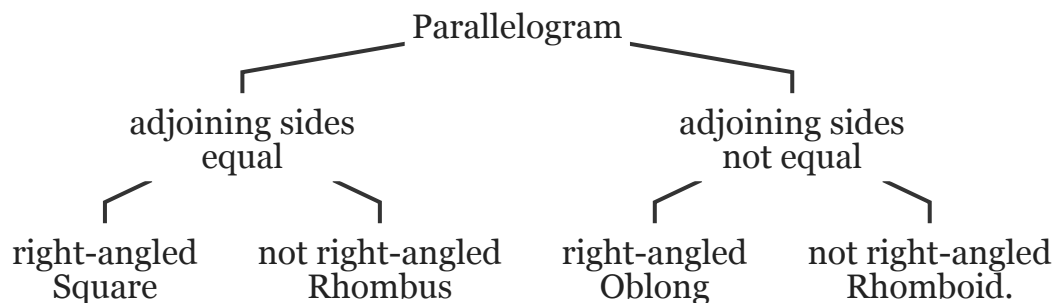
Logical division is the name of the process by which we distinguish the species of which a genus is composed. Thus we are said to divide the genus “book” when we consider it as made up of the groups folio, quarto, octavo, duodecimo books, &c., and the size of the books is in this case the ground, basis, or principle of division, commonly called the **Fundamentum Divisionis**. In order that a quality or circumstance may be taken as the basis of division, it must be present with some and absent with others, or must vary with the different species comprehended in the genus. A generic property of course, being present in the whole of the genus, cannot serve for the purpose of division. Three rules may be laid down to which a sound and useful division must conform:

1. The constituent species must exclude each other.
2. The constituent species must be equal when added together to the genus.
3. The division must be founded upon one principle or basis.

It would be obviously absurd to divide books into folio, quarto, French, German and dictionaries, because these species overlap each other, and there may be French or German dictionaries which happen to be quarto or folio and belong to three different species at once. A division of this kind is said to be a **Cross Division**, because there is more than one principle of division, and the several species in consequence cross each other and produce confusion. If I were to divide rectilinear figures into triangles, parallelograms, rectangles and polygons of more than four sides, I should commit all the possible faults in one division. The species parallelogram and rectangle do not exclude each other, since all rectangles must be parallelograms; the constituent species are not altogether equal to the genus rectilinear figure, since irregular four-sided figures which are not parallelograms have been omitted; and there are three principles of division, namely the number of sides, the directions of those sides, and the angles contained. But when subdivision is employed, and each of the species is considered as a genus which may be subjected to a further separation, a new principle of division may and in fact must be employed each time. Thus I can divide rectilinear figures according to the three principles mentioned above:



Here the principles of division are the number of their sides, and in the case of four-sided figures their parallelism. Triangles do not admit of division in this second respect. We may make a new division of parallelograms, adopting the equality of sides and the size of the angles as the principles; thus:



The most perfect divisions in a logical point of view are produced by continually dividing each genus into two

Now if we develop all the possible ways in which fungus, plant and animal can be combined together, we obtain for the term fungus—

- (1) Fungus, plant, animal.
- (2) Fungus, plant, not-animal.
- (3) Fungus, not-plant, animal.
- (4) Fungus, not-plant, not-animal.

Of these however the 4th cannot exist because by the premise a fungus must be a plant, or if not a plant an animal. The 1st and 3rd again cannot exist because the minor premise informs us that a fungus is not an animal. There remains then only the second combination,

Fungus, plant, not-animal,

from which we learn the syllogistic conclusion that “a fungus is a plant.”

The chief excellence of this mode of deduction consists in the fact that it is not restricted to any definite series of forms like the syllogism, but is applicable, without any additional rules, to all kinds of propositions or problems which can be conceived and stated. There may be any number of premises, and they may contain any number of terms; all we have to do to obtain any possible inference is to develop the term required into all its alternatives and then to examine how many of these agree with the premises. What remain after this examination necessarily form the description of the term. The only inconvenience of the method is that, as the number of terms increases, the number of alternatives to be examined increases very rapidly, and it soon becomes tedious to write them all out. This work may be abbreviated if we substitute single letters to stand for the terms, somewhat as in algebra; thus we may take *A, B, C, D*, &c., to stand for the affirmative terms, and *a, b, c, d*, &c., for the corresponding negative ones. Let us take as a first example the premises—

Organic substance is either vegetable or animal.

Vegetable substance consists mainly of carbon, hydrogen, and nitrogen.

Animal substance consists mainly of carbon, hydrogen, and nitrogen.

It would take a long time to write out all the combinations of the four terms occurring in the above; but if we substitute letters as follows—

A = organic substance,

B = vegetable substance,

C = animal substance,

D = consisting mainly of carbon, hydrogen, and nitrogen,

we can readily represent all the combinations which can belong to the term *A*.

(1)	<i>ABCD</i>	<i>AbCD</i>	(5)
(2)	<i>ABCd</i>	<i>AbCd</i>	(6)
(3)	<i>ABcD</i>	<i>AbcD</i>	(7)
(4)	<i>ABcd</i>	<i>Abcd</i>	(8)

Now the premises amount to the statements, that

A must be either B or C ,
 B must be D ,
 C must be D .

The combinations (7) and (8) are inconsistent with the first premise; the combinations (2) and (4) with the second premise; and (6) is inconsistent with the third premise. There remain only,

$ABCD$

$ABcD$

$AbCD$.

Whence we learn at once that “organic substance (A) always consists mainly of carbon, hydrogen and nitrogen,” because it always occurs in connexion with D . The reader may perhaps notice that the term $ABCD$ implies that organic substance may be both vegetable (B) and animal (C). If the first premise be interpreted as meaning that this is not possible, of course this combination should also be struck out. It is an unsettled point whether the alternatives of a disjunctive proposition can coexist or not (see p. 166), but I much prefer the opinion that they can; and as a matter of fact it is quite likely that there exist very simple kinds of living beings, which cannot be distinctly asserted to be vegetable only or animal only, but partake of the nature of each.

As a more complicated problem to shew the powers of this system, let us consider the premises which were treated by Dr Boole in his *Laws of Thought*, p. 125, as follows:

“Similar figures consist of all whose corresponding angles are equal, and whose corresponding sides are proportional.

Triangles whose corresponding angles are equal have their corresponding sides proportional; and *vice versa*.

Triangles whose corresponding sides are proportional have their corresponding angles equal.”

Now if we take our symbol letters as follows:

A = similar figure,

B = triangle,

C = having corresponding angles equal,

D = having corresponding sides proportional,

the premises will be seen to amount to the statements that

A is identical with CD ,

and that

BC is identical with BD ;

in other words, all A 's ought to be CD 's, CD 's ought to be A 's, all BC 's ought to be BD 's and all BD 's ought to be BC 's.

The possible combinations in which the letters may be united are 16 in number and are shewn in the following table:

<i>ABCD</i>	<i>aBCD</i>
<i>ABCd</i>	<i>aBCd</i>
<i>ABcD</i>	<i>aBcD</i>
<i>ABcd</i>	<i>aBcd</i>
<i>AbCD</i>	<i>abCD</i>
<i>AbCd</i>	<i>abCd</i>
<i>AbcD</i>	<i>abcD</i>
<i>Abcd</i>	<i>abcd</i>

Comparing each of these combinations with the premises we see that *ABCd*, *ABcD*, *ABcd*, and others, are to be struck out because every *A* is also to be *CD*. The combinations *aBCD* and *abCD* are struck out because every *CD* should also be *A*. Again, *aBCd* is inconsistent with the condition that every *BC* is also to be *BD*; and if the reader carefully follows out the same process of examination, there will remain only six combinations, which agree with all the premises, thus—

<i>ABCD</i>	<i>aBcd</i>
<i>AbCD</i>	<i>abCd</i>
	<i>abcD</i>
	<i>abcd</i>

From these combinations we can draw any description we like of the classes of things agreeing with the premises. The class *A* or similar figures is represented by only two combinations or alternatives; the negative class *a* or dissimilar figures, by four combinations, whence we may draw the following conclusion: “Dissimilar figures consist of all triangles which have not their corresponding angles equal, and sides proportional (*aBcd*), and of all